Provable Security for Program Obfuscation

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Outline

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Perfect Security

What do we want to get?

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We want to be sure that our system is safe to use.

In lecture 4 “Applications of Obfuscation” we’ll discuss what kind of safety we want to get by obfuscation.

Today: what does it mean to be sure about safety?

Usual approach: to build some proof of safety.

Ways to Security

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- Theoretic security: obfuscated program doesn't provide enough information to successful attack

  Example: exact reverse engineering. Solution: delete comments
- Computational (cryptographic) security: attack required too much computation

Necessary hardness of attack: average superpolynomial complexity.
Now: no problems with such proved complexity

Ways to Security II

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- NP-hard problems. Disadvantage: worst case complexity
- NP-hard problems with average complexity results.
  Example: SUBSET SUM
- Problems with wide-believed hardness:
  Examples: FACTORING, DISCRETE LOG

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- ⇨ Specific attacks on specific programs are computationally hard
- ⇨ For some classes of programs we can hide most of internal information
- ⇨ Some program analysis is proved to be hard
- ⇨ And obfuscation in general is impossible!

Ana and BAna

Slide from Lecture 1 — your turn to explain.

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We are interested in 2 types of polynomial-time analyzers:

- Ana is a source-code analyzer that can read the program.

$$Ana(P)$$

- BAna is a black-box analyzer that only queries the program as an oracle.

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Black-Box security

Ana can't get more information than BAna could

Black-Box Security: Formal Definition

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- **(functionality)** For every TM M, the string $O(M)$ describes the same function as M.
- **(polynomial slowdown)** The description length and running time of $O(M)$ are at most polynomially larger than that of M.
- **("virtual black box" property)** For any PPT A, there is a PPT S and a negligible function α such that for all TMs M

$$\left|Pr[A(O(M)) = 1] - Pr[S^M(1^{|M|}) = 1]\right| \leq \alpha(|M|).$$

Black-Box Security: Two Programs

A 2-TM obfuscator is defined in the same way as a TM-obfuscator, except the "virtual black box" property is changed as follows

- **("virtual black box" property)** For any PPT A, there is a PPT S and a negligible function α such that for all TMs M and N

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What obfuscator is more powerful?

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$$C_{\alpha,\beta}(x) = \begin{cases} \beta, & x = \alpha \\ 0, & \text{otherwise} \end{cases}$$

$$D_{\alpha,\beta}(C) = \begin{cases} 1, & C(\alpha) = \beta \\ 0, & \text{otherwise} \end{cases}$$

$$Z_k(x) = 0^k$$

Intuition: it is difficult to distinguish pairs $C_{\alpha,\beta}, D_{\alpha,\beta}$ from pair $Z_k, D_{\alpha,\beta}$ given only black box access to these programs.

Lemma Proof: Rough Sketch

Suppose O is 2-TM obfuscator. Let's check its "black box" property on pairs $C_{\alpha,\beta}, D_{\alpha,\beta}$ and $Z_k, D_{\alpha,\beta}$ for every α, β where $A = N(M)$.

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So we get a contradiction! But...

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There is a flaw in the proof. Do you see?

Impossibility Theorem

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$$F_{\alpha,\beta}(b,x) = C_{\alpha,\beta}\#D_{\alpha,\beta}$$

$$G_{\alpha,\beta}(b,x) = Z_k\#D_{\alpha,\beta}$$

Algorithm A is the following: to decompose M into two parts and evaluate the second part on the code (encoding) of the first.

Argument is similar to the Lemma's proof.

Property Hiding

Slide from Lecture 1 — your turn to explain.

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Instance: two families of programs Π_1 and Π_2

Adversary task: given a program $P \in \Pi_1 \cup \Pi_2$ to decide whether $P \in \Pi_1$ or $P \in \Pi_2$.

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Adversary task: given a program $P \in \Pi_1 \cup \Pi_2$ to decide whether $P \in \Pi_1$ or $P \in \Pi_2$.

Desirable protection: make adversary task as difficult as well-known computationally hard problem is.

Password Checking Hiding

prog π_1^w;
var x:string, y:bit;
input(x);
if $x = w$ then y:=1 else
y:=0;
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end of prog;

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prog π_0;
var x:string, y:bit;
input(x);
y:=0; output(y);
end of prog;

Task: Make this families indistinguishable.

Some Theoretical Background

One-Way Permutation is bijection from the set of all binary strings of length k to itself which is easy to compute and difficult to inverse.

$$F : B^k \to B^k$$

Hardcore Predicate for one way permutation F is a predicate (i.e. boolean function) h such that given $F(x)$ its difficult to predict $h(x)$ better than just guess it.

Usual construction of hard-core predicate: choose r by random and take any one way permutation F than given a pair $(F(x), r)$ its difficult to uncover $x \cdot r$.

Program with hidden password checking

```
prog Π
var x: string, y:bit;
const u, v:string, σ:bit;
input(x);
if ONE_WAY(x)=v then
      if x · u = σ then y:=1 else y:=0;
else y:=0;
output(y);
end of prog;
```

Model of Encrypted Computation

Slide from Lecture 1 — your turn to explain.

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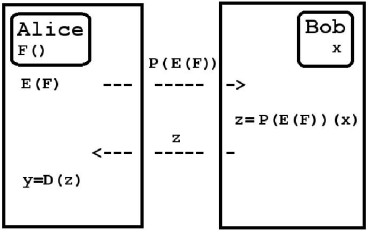

Basic task: keep F unknown to Bob.

Homomorphic Encryption

General idea: to design an encoding such that it is possible to evaluate various operations over encrypted messages (and getting encrypted results) without decrypting them.

In particular encoding is called

- ⇨ **Additively homomorphic** if it is possible to compute $E(x+y)$ from $E(x)$ and $E(y)$
- ⇨ **Multiplicatively homomorphic** if it is possible to compute $E(xy)$ from $E(x)$ and $E(y)$
- ⇨ **Mixed multiplicatively homomorphic** if it is possible to compute $E(xy)$ from $E(x)$ and y.

Homomorphic Encryption II

Fact: there exists additively homomorphic encryption schemes over the rings $\mathbb{Z}/N\mathbb{Z}$.

Corollary: there exists additively & mixed multiplicatively homomorphic encryption schemes over the rings $\mathbb{Z}/N\mathbb{Z}$.

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Proof: Mixed multiplication could be done by polynomial number of additions.

Encrypting Polynomials

Let P be polynomial over $\mathbb{Z}/N\mathbb{Z}$ ring.

$$P = \sum a_{i_1 \dots i_s} X_1^{i_1} \dots X_s^{i_s}$$

Then we can encrypt P by just sending encrypted coefficients (using MM-A homomorphic encryption). Bob is able to compute $E(P(X))$ and return it back to Alice.

What we reveal to Bob? Only set of nonzero coefficients of P.

Mobile cryptography results

What are further results for encrypted computation?

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⇨ Other presentations of function.
- [Loreiro, Molva] – function as a matrix.
- [Sander, Tschudin] – another basic hard problem: decomposition of rational functions.

More Black-Box Security

What are other functions obfuscated with black-box security?

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⇨ **[LPS 2004]** – interactive access control system.

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- [LPS 2004] – interactive access control system.
- Next results I expect from you!

Quality of obfuscating transformations

What is hard to get from programs after obfuscating transformations?

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- Alias analysis is NP-hard!
- Average hardness is proved only for several fixed analysis algorithms

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- ⇨ Obfuscation in general is impossible.

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- We can use cryptographic constructions to hide some internal constants.
- Obfuscation in general is impossible.

Question Time!

Not covered by the talk

- ⇨ Black box security with relations to zero-knowledge